Second Order Corrections to the Magnetic Moment of Electron at Finite Temperature

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Abstract

Magnetic moment of electron at finite temperature is directly related to the modified electron mass in the background heat bath. Magnetic moment of electron gets modified when it couples with the magnetic field at finite temperature through its temperature dependent physical mass. We show that the magnetic moment of electron becomes a complicated function of temperature and even change its temperature dependent behavior around the energies for primordial nucleosynthesis. We calculate the self-mass induced thermal contributions to the magnetic moment of electron, up to the two loop level, for temperatures valid around the era of primordial nucleosynthesis. A comparison of thermal behavior of the magnetic moment is also quantitatively studied in detail, around the temperatures below and above nucleosynthesis temperature range.

1 Introduction

Quantum Electrodynamics (QED) is well known as the simplest representative and the most accurate gauge theory. Thermal medium effects are incorporated in QED by taking into account the vacuum fluctuations for propagating particles along with the hot particles due to the background heat bath. For simplicity, all the particles in the background are assumed to be in thermal equilibrium with the heat bath. These particles are virtually created and annihilated continuously due the effects of heat bath at finite temperature. The interactions with the background particles (electrons, positrons and photons) are included through statistical distribution functions of fermions and bosons, known as Fermi-Dirac distribution for electrons and positrons and Bose-Einstein distribution for photons. These distribution functions represent the possibility of exchange of virtual particles with the real hot particles from the heat bath.

Electromagnetic interactions of particles get modified at finite temperature because many-body aspects of the statistical background are possessed by the hot medium. This replaces the notion of one-particle systems adopted for particle interactions in the vacuum.

The techniques for handling the particle interactions in the background medium have extensively evolved over the last few decades and are a part of standard literature [1]. It has been explicitly demonstrated [2-9] that in the hot background, the density-of-states factors have to be modified to include the real emission and absorption of particles which are in thermal equilibrium with the heat bath. The validity of the QED renormalization in a background of particles is refined by comparing the second order (in α) corrections with the first order corrections from the heat bath. It is explicitly seen, as expected, that one-loop radiative corrections are much larger than the two-loop corrections. Our scheme of calculations is based on the real part of the propagator and the results are fully acceptable, at least below the decoupling temperature, i.e., around 2 MeV.

Renormalization techniques of vacuum theory are extended to include finite temperature effects in a standard manner. Regular renormalization procedure for QED in vacuum is used at finite temperature to study the background effects on electron mass, charge and wave function renormalization constants. The modifications in the electromagnetic properties are estimated in terms of the renormalized values of QED parameters up to the two loop level [2-16].

Feynman rules at finite temperature remain the same as those in vacuum except that the particle propagators are appropriately modified. We work in Minkowski space where the Green's functions depend on real Minkowski momenta p^μ. Therefore, the dynamical processes such as particles propagating in the heat bath may be more conveniently dealt with. Moreover, in the real-time formulation, the thermal corrections can be easily kept separate from the vacuum corrections. Therefore, order by order cancellation of temperature dependent singularities, and the convergence of perturbative expansion can be straight away tracked down.

In this paper, thermal contributions to the anomalous magnetic moment of electron are specifically studied. The magnetic moment of electron is modified through the radiative corrections to the electron mass both at the one loop and two loop levels. We analyze net effect from the first order and the second order radiative contributions, including both from the irreducible and disconnected graphs up to the two loop level.

In the next section, we briefly present the calculations of thermal contributions to the physical mass of electron up to order α^2 through the radiative corrections at finite temperature [15], for completion. These self-mass corrections change the magnetic dipole moment of electron as a function of temperature at the two loop level also. Section 3 comprises of the calculation of the magnetic moment of electron up to the two loop level and analyzes these results in different ranges of temperatures. Section 4 is devoted to the discussion of the results of magnetic moment and some of the possible estimates of the results obtained in this work.

2 Self-Mass of an Electron at Finite Temperature

The renormalized mass of electron is represented by a physical mass given as

$$m_{phys} = m + \delta m. \tag{1}$$

where m is the electron mass at zero temperature. Radiatively corrected physical mass up to order α^2 is

$$m_{phys} \cong m + \delta m^{(1)} + \delta m^{(2)},$$

where $\delta m^{(1)}$ and $\delta m^{(2)}$ are the shifts in the electron mass at one and two loop level respectively. The physical mass is deduced by locating the pole of the propagator $\frac{i(\not p+m)}{p^2-m^2+i\varepsilon}$. For this purpose, all the finite terms in electron self-energy are combined together. Second order selfmass corrections due to the finite temperature (of the order α^2) are used here to estimate finite temperature effects on the magnetic moment of electron. The physical mass of the electron at one loop was obtained by writing

$$\Sigma(p) = A(p)E\gamma_{_0} - B(p)\vec{p}.\vec{\gamma} - C(p), \tag{2}$$

where $A(p)$, $B(p)$, and $C(p)$ are the relevant coefficients. Taking the inverse of the propagator with momentum and mass term separated as

$$S^{-1}(p) = (1-A)E\gamma^o - (1-B)p.\gamma - (m - C), \tag{3}$$

The temperature-dependent radiative corrections to the electron mass up to the first order in α, are obtained from the temperature modified propagator. These corrections are rewritten in the form of boson and fermion loop integrals at the one loop level as

$$\begin{aligned} E^2 - |\mathbf{p}|^2 &= m^2 + \frac{\alpha}{2\pi^2}(I.p + J_B.p + m^2 J_A) \\ &\equiv m^2_{phys}, \end{aligned} \tag{4}$$

where

$$I.p = \frac{4\pi^3 T^2}{3}, \tag{5}$$

and

$$J_B.p = 8\pi\left[\frac{m}{\beta}a(m\beta) - \frac{m^2}{2}b(m\beta) - \frac{1}{\beta^2}c(m\beta)\right]. \tag{6}$$

Thus up to the first order in α, thermal corrections to the mass of electron can be obtained as

$$m^2_{phys} = m^2\left[1 - \frac{6\alpha}{\pi}b(m\beta)\right] + \frac{4\alpha}{\pi}mT\ a(m\beta) + \frac{2}{3}\alpha\pi T^2\left[1 - \frac{6}{\pi^2}c(m\beta)\right]. \tag{7}$$

The first order correction at finite temperature is calculated as

$$\begin{aligned}\frac{\delta m}{m} &\simeq \frac{1}{2m^2}\left(m_{phys}^2 - m^2\right)\\ &\simeq \frac{\alpha\pi T^2}{3m^2}\left[1-\frac{6}{\pi^2}c(m\beta)\right]+\frac{2\alpha}{\pi}\frac{T}{m}a(m\beta)-\frac{3\alpha}{\pi}b(m\beta). \qquad (8)\end{aligned}$$

with $\frac{\delta m}{m}$ as the relative shift in electron mass due to finite temperature which was originally determined in Ref. [2] with

$$a(m\beta) = \ln(1+e^{-m\beta}), \qquad (9)$$

$$b(m\beta) = \sum_{n=1}^{\infty}(-1)^n \, \mathrm{Ei}(-nm\beta), \qquad (10)$$

$$c(m\beta) = \sum_{n=1}^{\infty}(-1)^n \frac{e^{-nm\beta}}{n^2}, \qquad (11)$$

At low temperature, the functions $a(m\beta)$, $b(m\beta)$, and $c(m\beta)$ fall off in powers of $e^{-m\beta}$ in comparison with $\left(\frac{T}{m}\right)^2$ and can be neglected so that

$$\frac{\delta m}{m} \overset{T\ll m}{\longrightarrow} \frac{\alpha\pi T^2}{3m^2}. \qquad (12)$$

Moreover, in the high-temperature limit, $a(m\beta)$ and $b(m\beta)$ are vanishingly small whereas $c(m\beta) \longrightarrow -\pi^2/12$, yielding

$$\frac{\delta m}{m} \overset{T>m}{\longrightarrow} \frac{\alpha\pi T^2}{2m^2}. \qquad (13)$$

Eq. (8) is valid for large temperatures relevant in QED including $T \sim m$. This range of temperature is particularly interesting from the point of view of primordial nucleosynthesis. It has been found that some parameters in the early universe such as the energy density and the helium abundance parameter Y become slowly varying functions of temperature [16] whereas they remain constant in both extreme limits given by $T \ll m$ and $T \gg m$.

Using the same procedure as the one used for one loop calculations, the relative shift in electron mass at the two loop level was obtained in Ref. [15]. This relative shift in electron mass introduces the temperature dependence in the magnetic moment of electron up to two loops. However, the two-loop order result is very complicated and cannot be easily simplified. Therefore, we will use the complete expression for the two loop calculations of electron selfmass, near the nucleosynthesis temperature, given in Ref [10-12]. In the following section, we compute the magnetic moment of electron from the self-mass of electron, up to the two loop level, in thermal background.

3 Magnetic Moment of Electron in the Heat Bath

The anomalous magnetic moment of an electron is generated due to the coupling of electron with the magnetic field through the radiative corrections. Some of these results are given in Ref. [16]. The electromagnetic coupling is affected by the electron mass and the radiative corrections to the electron mass. The coupling of electron mass with the external magnetic field is regulated through the mass of the particle itself. It is known from the calculation of the radiative corrections that the self-mass corrections to the electron are contributed by the distribution of hot bosons and fermions in the background medium. This effect, in turn, changes the electromagnetic properties of the medium itself. Therefore the magnetic moment gets changed with the temperature effects. The magnetic moment of electron is related to the relative shift in electron mass at finite temperature $\frac{\delta m}{m}$ as:

$$\mu_a = \frac{\alpha}{2\pi} - \frac{2}{3}\frac{\delta m}{m}. \tag{14}$$

The leading order contributions to the magnetic moment up to the one loop level is

$$\mu_a = \frac{\alpha}{2\pi} - \frac{2}{3}\alpha[\frac{\pi T^2}{3m^2}\{1 - \frac{6}{\pi}c(m\beta)\} + \frac{2}{\pi}\frac{T}{m}a(m\beta) - b(m\beta)] \tag{15}$$

which can be shown to be:

$$\mu_a = \frac{\alpha}{2\pi} - \frac{2}{9}\frac{\alpha\pi T^2}{m^2} \tag{16}$$

for $T < m$ while it becomes:

$$\mu_a = \frac{\alpha}{2\pi} - \frac{1}{3}\frac{\alpha\pi T^2}{m^2} \tag{17}$$

for $T > m$. First order in α contribution to μ_a around the temperature range relevant for primordial nucleosynthesis (i.e., $T \sim m$), soon after the big bang is given by expression in Eq. (15). The two loop contribution to the magnetic moment can simply be added to the magnetic moment in terms of the relative shift in electron mass $\frac{\delta m^{(2)}}{m}$ as

$$\mu_a = \frac{\alpha}{2\pi} - \frac{2}{3}(\frac{\delta m^{(1)}}{m} + \frac{\delta m^{(2)}}{m}). \tag{18}$$

Now using the expression for $\frac{\delta m}{m}$ at finite temperature in Ref. [12], we get thermal contributions to the magnetic moment up to the two-loop level as

$$
\begin{aligned}
\mu_a \;=\; & \frac{\alpha}{2\pi} - \;\frac{2}{3}\alpha[\frac{\pi T^2}{3m^2}\{1-\frac{6}{\pi}c(m\beta)\}+\frac{2}{\pi}\frac{T}{m}a(m\beta)-\frac{3}{\pi}b(m\beta)] \\
& -\frac{2\alpha^2}{3}[\frac{\pi T^2}{3m^2}\{1-\frac{6}{\pi}c(m\beta)\}+\frac{2}{\pi}\frac{T}{m}a(m\beta)-\frac{3}{\pi}b(m\beta)] \\
& \times[\frac{\pi T^2}{3m^2}\{1-\frac{6}{\pi}c(m\beta)\}+\frac{2}{\pi}\frac{T}{m}a(m\beta)-\frac{3}{\pi}b(m\beta)] \\
& -\frac{4}{3}\alpha^2\sum_{r=1}^{\infty}[T^2\{\sum_{n=3}^{r+1}\;(-1)^{n+r+1}\frac{\pi}{6mEv}\frac{e^{-\beta(rE+mn)}}{n} \\
& -\frac{3}{8}(-1)^r\frac{e^{-r\beta E}}{E^2v^2}[\frac{9E^2}{2m^2}+6\sum_{s=3}^{r+1}\frac{1}{s}+4\sum_{n,s=3}^{r+1}\frac{1}{ns}+(-1)^{s-r}\{\frac{9E}{m}(3+4\sum_{s=3}^{r+1}\frac{1}{s}) \\
& +2(\frac{E^2v^2}{m^2}-3)(9+18\sum_{s=3}^{r+1}\frac{1}{s}+8\sum_{n,s=3}^{r+1}\frac{1}{ns})\}]+\frac{4}{E^2v^2}\}-\frac{m^2\beta^2}{\pi^2}c(m\beta) \\
& -\frac{T}{m}\{\frac{\pi m}{6Ev}\sum_{s=2}^{r+1}\sum_{n=1}^{s+1}\frac{e^{-\beta(rE+mn)}}{n}[1\quad-\{(-1)^{r+n}-(-1)^{s+n}\}] \\
& +[\{\,\mathrm{Ei}(-m\beta)-\mathrm{Ei}(-2m\beta)\}\{\frac{9E}{4}(\frac{E}{E^2v^2}-\frac{1}{m})+(\frac{5E}{m}-21+\frac{E^2}{2m^2})\sum_{n=3}^{r+1}\frac{1}{n}\} \\
& +\{\frac{9}{4v^2}-\sum_{n=1}^{s+1}\sum_{s=3}^{r+1}[1-E^2(\frac{1}{2m^2}+\frac{3}{E^2v^2})+\frac{3E}{m}]\}(-1)^s\,\mathrm{Ei}\;(-sm\beta)] \\
& +\frac{e^{-rm\beta}}{m}\{[\frac{9E}{2v^2}+2(\frac{3E}{v^2}+\frac{3E^2v^2}{m}-5E)\sum_{n=3}^{r+1}\frac{1}{n}]\sum_{s=1}^{\infty}\sinh sm\beta \\
& -\frac{3m^3}{E^2v^2}(\frac{3}{4}-\sum_{n=3}^{r+1}\frac{1}{n})\sum_{s=1}^{\infty}\cosh sm\beta\}]\}+\frac{1}{m^2}\{\frac{9m}{4E^2v^2}(E^3+\frac{m^3}{2}) \\
& +[\frac{3m}{E^2v^2}(E^3+m^3)+5mE-3E^2v^2]\sum_{n=3}^{r+1}\frac{1}{n}\}\{\,\mathrm{Ei}(-m\beta)-2\,\mathrm{Ei}(-2m\beta)\} \\
& -\frac{1}{m^2}\sum_{n=3}^{r+1}\{\sum_{s=1}^{r+1}\frac{(-1)^s}{n}[\frac{m^2re^{-sm\beta}}{2} \\
& +\{sE(2m-\frac{E^2}{m})+\frac{m^2(s-r)}{2}\}\,\mathrm{Ei}(-sm\beta)] \\
& -\frac{\pi m^3}{3Ev}[e^{-\beta rE}\;(-1)^{n+r}(n+1)\quad-\sum_{s=2}^{r+1}(-1)^{n+s}]\,\mathrm{Ei}(-nm\beta)\}]. \qquad (19)
\end{aligned}
$$

It can be clearly seen from Eq. (19) that the second order corrections are suppressed by at least two orders of magnitudes as compared to the one loop contributions. Dependence of the self-mass induced thermal contributions to

the anomalous magnetic moment of electron is very complicated at the two loop level, as indicated by Eq. (19). The exact estimate of this magnetic moment for application to the primordial nucleosynthesis is very involved and probably is not really so significant at two loop level. However, the low temperature ($T < m$) and the high temperature ($T > m$) values of the magnetic moment can be quantitatively analyzed to prove the validity of the renormalization scheme. We give the plotting for magnetic moment μ_a vs $\frac{T}{m}$ in low temperature and the high temperature regions in the next section. This analysis indicates that the magnetic moment of electron changes its behavior around nucleosynthesis.

4 Results and Discussion

The electron mass acquires a significant contribution from the heat bath even for temperatures that are smaller than the electron mass. However, this dependence becomes very complicated as soon as the background temperature approaches the value of electron mass. One loop corrections to the electron mass at finite temperature are presented in Eq. (8). The low ($T < m$) and high ($T > m$) temperature values of self-mass of electron at the one loop level are given in Eqs. (12) and (13), respectively as limiting cases of Eq. (8). Incorporating the second order relation for the physical mass of electron [15] into Eq. (14) leads to Eq. (19) which gives the general form of the anomalous magnetic moment at finite temperature up to order α^2. When an electron couples with the magnetic field at finite temperature, a nonzero contribution to the magnetic moment is picked up due to the coupling of electron mass with the thermal background. Eq. (14) presents the acceptable relation of the magnetic moment with that of the self-mass of electron. The behavior of the magnetic moment of electron near the nucleosynthesis temperatures (Eq.(19)) is very complicated and it can be fitted through a single mathematical function under some special conditions only. However, we can extract the quantitative behavior of magnetic moment of electron for low and high temperatures in a comparatively simple form. Using previously studied second order contributions to the electron mass at low temperature ($T < m$), leading order contributions to the magnetic moment of electron can be computed as

$$\mu_a \xrightarrow{T<m} -\frac{2}{9}\frac{\alpha\pi T^2}{m^2} - 10\alpha^2\left(\frac{T^2}{m^2}\right) \tag{20}$$

whereas, the leading order contributions at high temperature ($T > m$) comes out to be

$$\mu_a \xrightarrow{T>m} -\frac{1}{3}\frac{\alpha\pi T^2}{m^2} - \frac{\alpha^2\pi^2}{6}\left(\frac{T^2}{m^2}\right)^2 + \frac{\alpha^2}{6}\frac{m^2}{T^2} \tag{21}$$

Eqs. (20) and (21) are used for a quantitative study of magnetic moment of electron at low temperature and high temperature, respectively. We plot the temperature dependence of magnetic moment of electron versus $\frac{T}{m}$. A plot of

Figure 1: Low temperature behavior of the magnetic moment of electron at the two loop level.

Eq. (20) is given in Fig. 1, whereas, Eq. (21) is plotted in Fig. 2. Both of these graphs give a sort of quadratic behavior in the negative sense. However, at high temperatures, the positive contribution of $(m/T)^2$ term gives a significant change in behavior.

Eqs. (21) and (22) and Figure 1 and Figure 2. indicate the difference between low temperature and high temperature behavior. Major difference in the behavior occurs due to the m^2/T^2 term. Contribution of this term reduces with increasing temperatures and becomes totally ignorable at very high temperatures. This is obvious from Figure 1 and Figure 2 that the magnetic moment of electron falls off rapidly with temperature after the nucelosynthesis as compared to that before the nucleosynthesis. This rapid decrease in magnetic moment after the nucleosynthesis is not the same as it is compared in Eqs. (16) and (17), implying that the one loop and two loop behaviors are not exactly similar.

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Figure 2: High temperature behavior of the magnetic moment of electron at the two loop level.

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